Welcome to ECE367
This class will introduce you to the fundamental theory and models of optimization as well as the geometry that underlies them. The first portion of the course focuses on geometry: recalling and generalizing linear algebraic concepts you first met in your linear algebra course. The second portion focuses on optimization. Presentation of applications is woven throughout. We will draw examples from diverse areas of the engineering and natural sciences. The material covered in this course will prove of interest to students from all areas of engineering, from the computer sciences and, more generally, from disciplines wherein mathematical structure and the use of numerical data is of central importance.

Prerequisites
The main prior courses that we will be building on are vector calculus and linear algebra.

Course Text
Lectures
Wednesdays: 4-5pm.
Thursdays: 3-5pm.
Per the course announcements, lectures will be delivered via Zoom. Highly interactive lectures are the most useful and engaging. Questions are welcomed and encouraged. We’ll do our best this term!

Tutorials
TUT0102: Mon 4:00pm, starting 14 September.
TUT0101: Wed 9:00am, starting 16 September.

Note, according to the official FASE timetable, this term ECE367 tutorials were assigned to be 2hrs in length. The purpose of this was to enable quizzes in tutorials. As we will be doing those quizzes online, tutorials will be shorter. Plan for 60-75minutes. One tutorial will be recorded and posted for asynchronous viewing (similar to the lectures) each week.

Office Hours
My office hours are an informal time to ask questions and learn from me and from each other. It’s better to discover early in the semester the usefulness of office hours, rather than to wait until just before a problem set is due or the final exam. As mentioned above my office hours will be held from 9-10am Thursday mornings via Zoom. Please see the Zoom invite distributed on 15 September.

Course Website
All course handouts will be posted on Quercus. Please make sure you can access. Also please let me know if you have not received course emailings (sent via Quercus). We need to have everyone on that list.

Discussion Board
We will be using Piazza (now integrated into Quercus) as the course discussion board. Please post all course-related question to the discussion board. It is quite likely others will have the same question and it is good to have the responses available to everyone. As is noted below in the “Class Participation” section, a portion of your course mark will be ties to your discussion board activities and the quality thereof. Details of assignments will follow.

As I mentioned in my message to the class the purpose of the discussion board is intended to foster intra-class discussion. While staff have access, and a TA will spend some time monitoring the board for topics of general interest / themes of discussion / points of confusion (observations that the TA will feed back to me), the TA time that I can allocate for posting to the board is quite limited. Thus, the board is not intended as a Q&A board for course staff. If you have specific technical questions you would like to ask of a staff member, we have time allocated for that. Please ask me after lecture (as you may have observed I will
“hang” around answering questions), in my office hours, or ask the TA in your tutorial or in the TA office hours.

If you have a personal issue, please contact one of the course staff (instructor, TA) directly.

Problem Sets

The course problem sets are designed to improve your understanding of the course material. They will include both written and numerical problems. In making up exams I will assume you have worked and/or studied the solutions to all problems.

The problem sets are a crucial part of the learning experience. They are designed to illustrate course concepts and material. Without working through and, perhaps, struggling at length with the problems, you will not develop as deep a facility with the concepts developed in class. Invariably this will have a major impact on the depth of your understanding and your final grade.

- **“THEORY” PARTS WILL NOT BE GRADED.** In the first offering of this course problem sets were handed in and were graded. As I handed out detailed solutions, that percentage of the grade will be re-allocated to regular quizzes. Problem sets continue to be a key, hands-on aspect of learning, both for the theoretical and implementational aspects of the course. Without first having suffered through trying to solve the problems you will not gain as much from looking through the solutions. Do not short-change yourself here.

- **“NUMERICAL” PORTION WILL BE GRADED:** Each problem set will contain certain numerical parts. These parts will be graded. For these you are asked to develop code. You will be asked to submit your code along with your solutions. While problems are designed for use with Matlab, e.g., through inclusion of sample snippets of code, you are welcome to use whatever language you prefer.

- **COLLABORATION:** Collaboration with one or (at most) two classmates on problem sets is encouraged. However, each student must individually write up their own solutions. If you do collaborate you must note on your solutions the names and student ID numbers of your collaborators. This will, e.g., help us understand the existence of similarity in your solution approach.

- **PROBLEM SET HAND-IN PROCEDURE:** We will follow up with details on this. The first problem set will not be due till the third week of the course.

- **LATE PROBLEM SETS** receive a 33.33% deduction for each 24 hours late or any portion thereof. This means 33.33% immediately after the due date, 66.66% if 24 hours late, and 100% if 48 hours. Solutions will be posted 48 hours after the due date.

The mathematical tools and perspectives we develop in the class find application in an enormous range of technologies. One objective of the problems sets is to introduce you to some of these diverse applications and to introduce you to standard numerical toolboxes.
(e.g., CVX) that are used in some of these technologies. As examples, below we list some of the possible numerical exercises for the problem set in which you will work with the concepts developed in class.

- **Vector spaces and functions:** Word vectors, angles, and inverse document frequency; first-order Taylor approx; discrete time Fourier series (approximating a square wave); projection with different norms; Taylor vs. $\ell_2$ approximations.
- **Matrices and Eigen decomposition:** Second-order Taylor approximations; Google’s PageRank algorithm.
- **SVD and PCA:** Latent semantic indexing (and word-vec); Eigenfaces and $\ell_2$ projection.
- **Least squares:** Optimal control under $\ell_2$; computer-aided tomography (CAT scanning).
- **LPs and QPs:** Various LPs and QPs; optimal control under $\ell_1$ and $\ell_\infty$; Markowitz portfolio optimization.
- **Image coding:** Sparse coding of images.

**Course Grade**

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Problem sets:</td>
<td>15%</td>
</tr>
<tr>
<td>Quizzes:</td>
<td>12%</td>
</tr>
<tr>
<td>Class Participation:</td>
<td>8%</td>
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<tr>
<td>Midterm:</td>
<td>25%</td>
</tr>
<tr>
<td>Final exam:</td>
<td>40%</td>
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</table>

- **Quizzes:** There will be six (6) quizzes, roughly every-other week. Quizzes will be deployed using the Quercus quiz functionality. Quizzes will occur in the second half of your assigned tutorial section.
- **Midterm:** The midterm will be held on Thursday, 29 October, starting at 8pm Toronto time.
- **Final exam:** The final exam will be scheduled in the end-of-semester exam period.
Class Participation (w/ thanks to Prof. Chan Carusone for allowing use of his rubric)

Discussion boards are an important part of online learning. In this course, they will be used to evaluate your understanding of the course material and your ability to communicate and work with peers.

There are two parts to your discussion board mark. A rubric for each is provided below:

Part I, General participation (5% of final grade): includes participation in all discussion boards and threads:

- Excellent (5): 5+ posts asking detailed questions about lecture material, sharing outside source material that is relevant to the topics covered in the lectures or problem sets, providing answers to peer questions about lectures, homework or labs.

- Good (4): Multiple posts asking detailed questions about lectures or problem sets plus relatively few and/or superficial posts regarding lecture material.

- Weak (3): A few posts made in passing through the course.

- Poor (0-2): Little or no serious contribution to the discussion board.

Part II, Assigned facilitation (3% of final grade): during the semester you will each be assigned one particular lecture, problem set, or numerical exercises for which you must facilitate the discussion

- Excellent (3): A detailed and clear summary post making connections to other material in the course, that helps others get going on the material, followed by additional prompt moderating replies to any subsequent discussion on your assigned lecture or problem set.

- Good (2): A clear summary post followed by replies to subsequent discussion on your assigned materials.

- Weak (1): A modest summary and superficial follow-up replies.

- Poor (0): A brief or difficult to understand summary with superficial or missing follow-up replies.

Your ability to communicate clearly via threaded discussions is an essential skill.

- Posts that are for any reason difficult to understand, out-of-context, or disruptive to the discussion will be graded accordingly. Give your posts very specific and descriptive headings. This will help everyone navigate the discussion boards.

- Use threaded discussions to organize posts by specific topics.

- Subscribe to the discussion boards so you are notified about the activity.

- The “assigned facilitation” assignments will be forthcoming.
Reference Texts

There are a number of reference texts for this course. The first three are available in electronic form on their respective authors’ websites (in the case of the second text in a condensed form).

1. *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares* by Stephen Boyd and Lieven Vandenberghe, Cambridge University Press, 2018. This is an introductory linear algebra text. It is a good place to look to to review your exposure to linear algebra in MAT185/188. It also introduces some of the topics we touch on that (I believe) were not covered in MAT185/188, e.g., non-Euclidean vector norms. We will assign some problems from this textbook. They are accessible via the online version.

2. *Introduction to linear algebra* by Gilbert Strang, Wellesley-Cambridge Press. Now in its 6th edition. This is a classic text, it’s the text I used to learn linear algebra, and Strang is a great teacher. See the book’s website for lots of online resources: http://math.mit.edu/~gs/linearalgebra/.

3. *Linear Algebra Done Right*, by Sheldon Axler, Springer Press, 2015. This is a text for a “second course” in linear algebra. It develops linear algebra for finite-dimensional vector spaces in a fully mathematical manner. It is a great and easy-to-read textbook that I highly recommend. Results we don’t prove in course are proved here. If you are motivated to understand the linear algebraic portion of the course even more fundamentally, you will like this text. If you (eventually) go to graduate school in this area I recommend you get a copy (now or later). If you do decide to get a hard-copy you may want to look for the second edition (the 2015 edition is the third). While somewhat less material is covered in the second edition, the formatting is less rich and therefore (in my opinion) is nicer and easier to read. Please note that an electronic version of a condensed version the third edition of this text available on the author’s website.

4. *Convex Optimization*, by Stephen Boyd and Lieven Vandenberghe, Cambridge University Press, 2004. This is the course text used in ECE1505, the graduate convex optimization course. Look here for deeper discussion of many of the optimization concepts developed in this course. An electronic version of this text is available on the authors’ websites.

If you want to delve even further into connections between linear algebra and optimization, the following are classic references.

5. *Nonlinear Programming*, by Dimitri P. Bertsekas, Athena Scientific, 1999. This is a widely used alternative text to Boyd and Vandenberghe’s *Convex Optimization* text.

7. *Convex Analysis*, R. Tyrell Rockafellar, Princeton University press, 1997. This reprint of the original is the place to go to see the story of convexity from a more mathematical perspective. Beautiful proofs.
Topics, Tentative Syllabus, and Readings

A list of course topics is provided below. In brief, we aim to work through the first ten chapters of the text in depth.

- Introduction (Chapter 1)
- Linear algebra:
  - Vectors and functions (Chapter 2)
  - Matrices (Chapter 3)
  - Symmetric matrices (Chapter 4)
  - Singular value decomposition (Chapter 5)
  - Linear equations and least squares (Chapter 6)
- Convex optimization models:
  - Linear, quadratic, and geometric models (Chapter 9)
  - Second-order cone and robust models (Chapter 10)
  - Convexity (Chapter 8)
- Further topics*
  - Matrix algorithms (Chapter 7)
  - Semidefinite models (Chapter 11)
  - Introduction to algorithms (chapter 12)
  - Applications (Chapters 13-16)

*Depending on how we progress through the term we will optionally cover materials from these chapters, sometimes in lecture, sometimes in tutorial, sometimes in the problem sets.
# Detailed Syllabus from 2018 (Part I – first half of course)

<table>
<thead>
<tr>
<th>#</th>
<th>hrs</th>
<th>date</th>
<th>ch.</th>
<th>out</th>
<th>in</th>
<th>topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>09/06</td>
<td>R</td>
<td>1,2</td>
<td></td>
<td>Initial handout review, vectors spaces, norms, start of inner products.</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>09/11</td>
<td>T</td>
<td>2</td>
<td>P1</td>
<td>Inner products, orthogonal decomposition.</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>09/13</td>
<td>R</td>
<td>2</td>
<td></td>
<td>Projection onto subspaces, affine sets, hyperplanes, non-Euclidean projection.</td>
</tr>
<tr>
<td>T1</td>
<td>09/10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Gradients and geometry of Taylor approx.</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>09/18</td>
<td>T</td>
<td>2</td>
<td></td>
<td>Func., graphs, epigraphs, level sets, half-spaces.</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>09/20</td>
<td>R</td>
<td>3</td>
<td></td>
<td>Matrices, matrix mult, vec space of matrices, inverse &amp; pseudo-inverse, orthogonal matrix, Jacobian, 2nd order Taylor approx.</td>
</tr>
<tr>
<td>T2</td>
<td>09/17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\ell_2$ projection: 2D subspace, affine set, hyperplane.</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>09/25</td>
<td>T</td>
<td>3</td>
<td>P2</td>
<td>P1</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>09/27</td>
<td>R</td>
<td>3</td>
<td></td>
<td>Nullspace; interesting directions; PageRank, power iteration method</td>
</tr>
<tr>
<td>T3</td>
<td>09/24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Geometry of the determinant.</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>10/02</td>
<td>T</td>
<td>3</td>
<td></td>
<td>Recap PageRank and power iteration method, PageRank example with $\text{GM}(\lambda = 1) = 2$; algebraic and geom. multiplicities; solving for eigenvalue and eigenvectors; $A = \begin{bmatrix} 2 &amp; 1 \ 0 &amp; 1 \end{bmatrix}$.</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>10/04</td>
<td>R</td>
<td>4</td>
<td></td>
<td>Example $A = \begin{bmatrix} 1 &amp; 1 \ 0 &amp; 1 \end{bmatrix}$; recap of terminology; diagonalization, symmetric matrices, quad. func., Taylor approx of quadratics, spectral theorem, variational characterization of eigenvalues.</td>
</tr>
<tr>
<td>T4</td>
<td>10/01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Examples of computing range- and null-spaces of $A$ and $A^T$, eigenvalues and eigenvectors.</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>10/09</td>
<td>T</td>
<td>4</td>
<td>P3</td>
<td>P2</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>10/11</td>
<td>R</td>
<td>4</td>
<td></td>
<td>Geometry of sample mean/covariance; sq-root and Cholesky; intro to SVD; first half of SVD proof (OptM Thm 5.1)</td>
</tr>
<tr>
<td>T5</td>
<td>10/10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Recap of symmetric matrices and spectral decomposition, plotting ellipsoids.</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>10/16</td>
<td>T</td>
<td>5</td>
<td></td>
<td>Finish SVD development (OptM Thm 5.1), calculating SVDs</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>10/18</td>
<td>R</td>
<td>5</td>
<td></td>
<td>PCA, geometry: rotate-scale+redimension-rotate, basis for null and range spaces from SVD</td>
</tr>
<tr>
<td>T6</td>
<td>10/17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Computing SVDs; low-rank matrix approx(OptM 5.1 and Ex 5.2).</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>10/23</td>
<td>T</td>
<td>5</td>
<td>P4</td>
<td>P3</td>
</tr>
</tbody>
</table>
**Detailed Syllabus from 2018 (Part II – second half of course)**

<table>
<thead>
<tr>
<th>#</th>
<th>hrs</th>
<th>date</th>
<th>ch.</th>
<th>out</th>
<th>in</th>
<th>topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2</td>
<td>10/25</td>
<td>R</td>
<td>6</td>
<td></td>
<td>Linear equations and least squares</td>
</tr>
<tr>
<td>T7</td>
<td></td>
<td>10/24</td>
<td></td>
<td></td>
<td></td>
<td>Midterm review.</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>10/30</td>
<td>T</td>
<td>6</td>
<td></td>
<td>Ex: linear regression, system ID, weighted LS.</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>11/01</td>
<td>R</td>
<td></td>
<td></td>
<td>MIDTERM (no class).</td>
</tr>
<tr>
<td>T8</td>
<td></td>
<td>10/31</td>
<td></td>
<td></td>
<td></td>
<td>Examples of least squares problems.</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>11/06</td>
<td>T</td>
<td>6</td>
<td></td>
<td>Regularized least squares, Tikhanov regular., and visualization of the geometry</td>
</tr>
<tr>
<td>19</td>
<td>2</td>
<td>11/08</td>
<td>R</td>
<td>9</td>
<td>P5</td>
<td>P4 Linear programs: definitions, geometry of objective function and constraints, examples, optimality condition, Simplex alg.</td>
</tr>
<tr>
<td>T9</td>
<td></td>
<td>11/07</td>
<td></td>
<td></td>
<td></td>
<td>The geometry of linear programs.</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>11/13</td>
<td>T</td>
<td>9</td>
<td></td>
<td>Remarks, $\ell_1$ and $\ell_\infty$ problems as LPs, intro to QPs, example of QPs: LS, linearly constrained quadratic minimization.</td>
</tr>
<tr>
<td>21</td>
<td>2</td>
<td>11/15</td>
<td>R</td>
<td>9</td>
<td></td>
<td>QPs: finish linearly constrained QPs, Markowitz portfolio optimization, geometry of the objective, active set algorithms for solving QPs.</td>
</tr>
<tr>
<td>T10</td>
<td></td>
<td>11/14</td>
<td></td>
<td></td>
<td></td>
<td>Examples of LPs.</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>11/20</td>
<td>T</td>
<td>8</td>
<td>P5</td>
<td>P6 QCQPs, intro to convexity, linear/affine/convex/conic hulls and sets</td>
</tr>
<tr>
<td>23</td>
<td>2</td>
<td>11/22</td>
<td>R</td>
<td>8</td>
<td></td>
<td>Operations that preserve convexity, important sets (norm balls, ellipsoids) and cones ($S_+^n$), convex functions, convex functions and sets, sublevel sets, quasi-convexity, transformations, conditions for convexity (line, 1st, 2nd order)</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>11/27</td>
<td>T</td>
<td>8</td>
<td></td>
<td>More on the 3 conditions for convexity, examples, max of a set of functions, function composition.</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>11/29</td>
<td>R</td>
<td>8</td>
<td></td>
<td>Convex optimization problems, local and global minima, conditions for global optimality.</td>
</tr>
<tr>
<td>T12</td>
<td></td>
<td>11/28</td>
<td></td>
<td></td>
<td></td>
<td>Course review.</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
<td>12/04</td>
<td>T</td>
<td>12</td>
<td>P6</td>
<td>Algorithms (gradient descent and Newton’s), course wrap-up.</td>
</tr>
</tbody>
</table>
Notice of video recording & sharing: Download permissible; re-use prohibited

At times during this course, some interactions including your participation, may be recorded on video and will be available to students in the course for viewing remotely after each session.

Course videos and materials belong to your instructor, the University, and/or other source depending on the specific facts of each situation, and are protected by copyright. In this course, you are permitted to download session videos and materials for your own academic use, but you should not copy, share, or use them for any other purpose without the explicit permission of the instructor. For questions about recording and use of videos in which you appear please contact your instructor.

Academic Honesty

You should already be familiar with the University of Toronto’s academic honesty policy (the “Code of Behavior on Academic Matters”) that deals with issues including plagiarism and cheating. Note that, as should be obvious, plagiarism on problem sets is plagiarism. As noted in the section on problem sets: “Collaboration with one or two classmates is encouraged. However, each student must individually write up their own solutions. Please note on your solutions the names of your collaborators.” Using other resources, such as getting your answers from another student or finding them online, rather than working them out yourself, is plagiarism. For a review of the policy please navigate to the following links:

http://www.academicintegrity.utoronto.ca/


Inclusivity, Accommodations and Mental Health Support

Statement on Inclusivity

You belong here. The University of Toronto commits to all students, faculty and staff that you can learn, work and create in a welcoming, respectful and inclusive environment. In this class, we embrace the broadest range of people and encourage their diverse perspectives. This team environment is how we will innovate and improve our collective academic success. You can read the evidence for this approach here.

We expect each of us to take responsibility for the impact that our language, actions and interactions have on others. Engineering denounces discrimination, harassment and unwelcoming behaviour in all its forms. You have rights under the Ontario Human Rights Code. If you experience or witness any form of harassment or discrimination, including but not limited to, acts of racism, sexism, Islamophobia, anti-Semitism, homophobia, transphobia, ableism and ageism, please tell someone so we can intervene. Engineering takes these reports extremely seriously. You can talk to anyone you feel comfortable approaching, including your
professor or TA, an academic advisor, our Assistant Dean, Diversity, Inclusion and Professionalism, the Engineering Equity Diversity and Inclusion Action Group, any staff member or a U of T Equity Office.

You are not alone. Here you can find a list of clubs and groups that support people who identify in many diverse ways. Working together, we can all achieve our full potential.

Statement on Accommodations

The University of Toronto supports accommodations for students with diverse learning needs, which may be associated with mental health conditions, learning disabilities, autism spectrum, ADHD, mobility impairments, functional/fine motor impairments, concussion or head injury, blindness and low vision, chronic health conditions, addictions, deafness and hearing loss, communication disorders and/or temporary disabilities, such as fractures and severe sprains, or recovery from an operation.

If you have a learning need requiring an accommodation the University of Toronto recommends that students register as soon as possible with Accessibility Services.
Phone: 416-978-8060
Email: accessibility.services@utoronto.ca

Statement on Mental Health

As a university student, you may experience a range of health and/or mental health challenges that could result in significant barriers to achieving your personal and academic goals. Please note, the University of Toronto and the Faculty of Applied Science & Engineering offer a wide range of free and confidential services that could assist you during these times.

As a U of T Engineering student, you have an Academic Advisor (undergraduate students) or a Graduate Administrator (graduate students) who can support you by advising on personal matters that impact your academics. Other resources that you may find helpful are listed on the U of T Engineering Mental Health & Wellness webpage, and a small selection are also included here:

- Accessibility Services & the On-Location Advisor
- Graduate Engineering Council of Students’ Mental Wellness Commission
- Health & Wellness and the On-Location Health & Wellness Engineering Counsellor
- Inclusion & Transition Advisor
- U of T Engineering Learning Strategist and Academic Success
- My Student Support Program (MySSP)
- Registrar’s Office
- SKULE Mental Wellness
- Scholarships & Financial Aid Office & Advisor
If you find yourself feeling distressed and in need of more immediate support resources, consider reaching out to the counsellors at My Student Support Program (MySSP) or visiting the Feeling Distressed webpage.