Broadband Electromagnetic and Stochastic Modelling and Signal Analysis of Multiconductor Interconnections

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Overview

- introduction
- RLGCG-modelling of multiconductor lines
- variability analysis along the signal propagation direction
- analysis of statistical signals resulting from random variations in geometry, material properties, component values, linear and non-linear drivers and loads
- brief conclusions
- questions & discussion
How to model signal integrity?

- **full 3D numerical tools**
  - direct access to multiport S-parameters and time-domain data
  - holistic but time-consuming
  - (some times too easily) believed to be accurate

- **divide and conquer**
  - multiconductor lines, vias, connectors, packages, chips, ….
  - model each of them with a dedicated tool
  - derive a circuit model for each part
  - obtain the S-parameters and time-domain data (eye-diagram, BER, crosstalk) from overall circuit representation
  - gives more insight to the designer (optimisation)
  - overall accuracy might be difficult to assess
Multiconductor Transmission Lines

simplify (idealize) to a 2D problem

2D fields, charges, currents

PlayStation 3 motherboard

3D fields, charges, currents

Transmission lines voltages & currents

RLGC
Telegrapher’s equations (RLGC)

\[
\frac{\partial \mathbf{i}}{\partial z} = -j\omega \tilde{\mathbf{C}} \mathbf{v} \quad \tilde{\mathbf{C}} = \mathbf{C} + \mathbf{G} / j\omega \\
\frac{\partial \mathbf{v}}{\partial z} = -j\omega \tilde{\mathbf{L}} \mathbf{i} \quad \tilde{\mathbf{L}} = \mathbf{L} + \mathbf{R} / j\omega
\]

N+1 conductors one of which
plays the role of reference conductor

\( \mathbf{i} \): Nx1 current vector
\( \mathbf{v} \): Nx1 voltage vector
\( \mathbf{C} \): NxN capacitance matrix
\( \mathbf{L} \): NxN inductance matrix
\( \mathbf{G} \): NxN conductance matrix
\( \mathbf{R} \): NxN resistance matrix
Multiconductor TML on-chip interconnect example:

- 4 differential line pairs
- semi-conducting region
- unusual reference conductor

**wish list number 1**

- broadband results (time-domain)
- many regions (some semi-conducting)
- good conductors (e.g. copper)
- small details
- exact current crowding and skin effect modelling

**wish list number 2**

- variability in cross-section
- variability along propagation direction
- stochastic responses
- prediction of stochastics for overall interconnects (sources, via’s, lines, ..)
The manufacturing process introduces **variability** in the geometrical and material properties but also along the signal propagation direction.

**Photolithography:** trace separation

**Impurities:** permittivity, loss tangent, etc.

Deterministic excitations produce **stochastic responses**.

**Random parameters**
- introduction
- RLGCG-modelling of multiconductor lines
- variability analysis along the signal propagation direction
- analysis of statistical signals resulting from random variations in geometry, material properties, component values, linear and non-linear drivers and loads
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- questions & discussion
C and G can be found by solving a classical potential problem in the cross-section:

- sources/unknowns: (equivalent) boundary charges
- preferred method: boundary integral equation
- relation between total charges and voltages $Q = C V$

$L$ and $R$ could be found by determining the magnetic fields due to equivalent contrast currents $j_{eq,z} = (\sigma + j\omega(\epsilon - \epsilon_0)) e_z$ placed in free space

Suppose we find a way to replace these currents by equivalent ones on the boundaries:

- sources/unknowns: equivalent boundary currents
- preferred method: EFIE with $e_z = -j\omega A_z - \frac{\partial \phi}{\partial z}$ on conductor $-j\omega A_z - j\omega Li$
two non-magnetic media “out” & “in”  
(conductor, semi-conductor, dielectric)  
separated by surface S  
fields inside $E_1, H_1$  
fields outside $E_0, H_0$

we introduce a fictitious (differential) surface current $J_s$  
a single homogeneous medium “out”  
fields inside differ: $E, H$  
fields outside remain identical: $E_0, H_0$
Advantages

- modelling of the volume current crowding / skin-effect is avoided
- less unknowns are needed (volume versus surface)
- homogeneous medium: simplifies Green’s function
- valid for all frequencies
- losses from DC to skin effect + “internal” inductance can all be derived from \( J_s \) and \( E_{\tan} \) on \( S \)

Disadvantage or Challenge 😊

The sought-after \( J_s \) is related to \( E_{\tan} \) through a non-local surface admittance operator

\[
J_s(r) = \int_S \mathcal{Y}(r, r') E_{\tan}(r') dS(r') \quad \text{in 3D}
\]

\[
j_{s,z}(r) = \int_c \mathcal{Y}(r, r') e_z(r') dc(r') \quad \text{in 2D}
\]

admittance operator similar to \( j_z(r) = \sigma \ e_z (r) \) but no longer purely local !
Differential Admittance

in 2D

\[ j_{s,z}(r) = \int_{c} \mathcal{Y}(r, r') e_z(r') dc(r') \]

- analytically using the Dirichlet eigenfunctions of S

- numerically for any S using a 2D integral equation (prof. P. Triverio)

in 3D

\[ J_S(r) = \int_{S} \mathcal{Y}(r, r') E_{tan}(r') dS(r') \]

- analytically using the solenoidal eigenfunctions of the volume V

- see e.g. Huynen et al. AWPL, 2017, p. 1052
Admittance operator

\[ j_{c,z}(\mathbf{r}) = \int_{c_n} \mathcal{Y}(\mathbf{r}, \mathbf{r}') e_z(\mathbf{r}) d\mathbf{c}(\mathbf{r}') \]

- 79.1 MHz - skin depth \( \delta = 7.43 \, \mu m \)
- 10 GHz - skin depth \( \delta = 0.66 \, \mu m \)

\( \mathcal{Y}_{s,local} = \sqrt{\frac{j \omega \mu}{\sigma}} \)
Telegrapher’s equations (RLGC)

\[
\frac{\partial i}{\partial z} = -j\omega \tilde{C} v \\
\frac{\partial v}{\partial z} = -j\omega \tilde{L} i
\]

\[\tilde{C} = C + G/j\omega\]
\[\tilde{L} = L + R/j\omega\]

**Final result:**

The 2-D per unit of length (p.u.l.) transmission line matrices \( R, L, G, \) and \( C \), as a function of frequency

(see ref. [5])

**wish list number 1**

- broadband results 😊
- many regions (some semi-conducting) 😊
- good conductors (e.g. copper) 😊
- small details 😊
- exact skin effect modelling 😊
$\varepsilon_r = 3.2$

$\sigma_{\text{copper}} = 5.8 \times 10^7$

$\tan\delta = \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} = 0.008$

$\tilde{C}_{11} = \tilde{C}_{22} = (96.16 - j0.64)\text{pF/m}$ and $\tilde{C}_{12} = \tilde{C}_{21} = (-4.19 + j0.002)\text{pF/m}$
Differential line pair

\[ L_{11} = L_{22} \]
\[ L_{12} = L_{21} \]
\[ R_{11} = R_{22} \]
\[ R_{12} = R_{21} \]
Metal Insulator Semiconductor (MIS) line

\[ \varepsilon_r = 9.7, \sigma = 0 \]

\[ \varepsilon_r = 9.7, \sigma \]

\[ \sigma = 50\text{S/m} \]

\[ L_{DC} = 422.73\text{nH/m} \]

\[ C_{DC} = 481.71\text{pF/m} \]
Metal Insulator Semiconductor (MIS) line @ 1GHz

Examples

$\tan \delta = \frac{\sigma}{\omega \varepsilon}$

$\varepsilon_r = 9.7, \sigma = 0$

$\varepsilon_r = 9.7, \sigma$

$v_{\text{diesel}} = \frac{c}{\sqrt{\varepsilon_r}} = 3.10^8 / \sqrt{9.7} \ m/s$

$10^{-2} < \tan \delta < 10^7$

$0.0054 \ S/m < \sigma < 5.4 \times 10^6 \ S/m$

good dielectric
good conductor
Coated submicron signal conductor

copper: 1.7 $\mu\Omega$cm
chromium: 12.9 $\mu\Omega$cm
coating thickness $\delta$: 10 nm
Coated submicron signal conductor

Inductance and resistance p.u.l as a function of frequency
Pair of coupled inverted embedded on-chip lines

Aluminum: \( \sigma = 3.77 \times 10^7 \text{ S/m} \)

Silicon: \( \varepsilon_r = 11.7, \sigma = 10 \text{ S/m} \)

\( \varepsilon_r = 3.9, \tan \delta = 0.001 \)
Pair of coupled inverted embedded on-chip lines

Discretisation for solving the RLGC-problem
Pair of coupled inverted embedded on-chip lines: L and R results
Pair of coupled inverted embedded on-chip lines: G and C results
Examples

4 differential pairs on chip interconnect

+ all dimensions in $\mu$m
+ $\sigma_{\text{sig}} = 40\text{MS/m}$
+ $\sigma_{\text{sub}} = 2\text{S/m}$
+ $\sigma_{\text{dop}} = 0.03\text{MS/m}$

$\epsilon_0$

$4\epsilon_0$

$0.4\mu\text{m}$

$384$

$300\mu\text{m}$

PEC
Examples
Electromagnetics Group

eight quasi-TM modes

quasi-even

mode $m_1$ (SWF = 2.41)

mode $m_2$ (SWF = 2.67)

mode $m_3$ (SWF = 3.08)

mode $m_4$ (SWF = 2.85)

the modal voltages $V = V_0 \exp(-j\phi)$ are displayed ($V_0 = \square$) @ 10GHz

quasi-odd

mode $m_5$ (SWF = 4.17)

mode $m_6$ (SWF = 4.24)

mode $m_7$ (SWF = 4.29)

mode $m_8$ (SWF = 4.28)

slow wave factor:
mode prop. velocity $v = c$/SWF
complex capacitance matrix @10GHz

Capacitance \( \tilde{C} = C + G/j\omega \)

\( (pF/m) \)
complex inductance matrix @10GHz

Inductance $\tilde{L} = L + R/j\omega$  
($\mu H/m$)

$L_{44}$  
$L_{33}$  
$L_{34}$

$R_{33}/\omega$  
$R_{44}/\omega$  
$R_{34}/\omega$

Substrate loss tangent $\sigma_{sub}/\omega\epsilon_{sub}$
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What if the cross-section varies along the propagation direction?

use a perturbation approach!

Quick illustration for a single line (with $L$ & $C$ complex – hence $R$ & $G$ are included)

\[
\begin{align*}
\frac{dV(z)}{dz} &= -j\omega L(z)I(z) \\
\frac{dI(z)}{dz} &= -j\omega C(z)V(z)
\end{align*}
\]

+ perturbation around nominal value

\[
\begin{align*}
\frac{d^2 \tilde{V}}{dz^2} + k_0^2 \tilde{V} &= 0 \\
\frac{d^2 \Delta V_1}{dz^2} + k_0^2 \Delta V_1 &= -k_0^2 \tau_C \tilde{V} - jk_0 \frac{d}{dz}(\tau_L Z_0 \tilde{I}) \\
\frac{d^2 \Delta V_2}{dz^2} + k_0^2 \Delta V_2 &= -k_0^2 \tau_C \Delta V_1 - jk_0 \frac{d}{dz}(\tau_L Z_0 \Delta I_1)
\end{align*}
\]

nominal

perturbation step 1

perturbation step 2

including this second order is CRUCIAL!

\[
\begin{align*}
k_0 &= \omega \sqrt{\tilde{L}} \tilde{C} \\
\tau_C &= (\Delta C)/(\tilde{C}) \\
Z_0 &= \sqrt{\tilde{L}/\tilde{C}} \\
\tau_L &= (\Delta L)/(\tilde{L})
\end{align*}
\]
Fibre weave: differential stripline pair on top of woven fiberglass substrate

cross-section of differential stripline pair
Fibre weave - discretisation (in CAD tool)

cross-section a

cross-section b
Fibre weave - material properties

real part of dielectric permittivity $\varepsilon'_r$ and $\tan\delta$ as a function of frequency
Propagation characteristics for a 10 inch line

differential mode transmission

forward differential to common mode conversion
- introduction
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- PART 1: MTL
- brief conclusions
- questions & discussion
- Interconnect designers need to perform statistical simulations for variation-aware verifications

- Virtually all commercial simulators rely on the Monte Carlo method
  - Robust, easy to implement 😊
  - Time consuming:
    slow convergence $\sim 1/\sqrt{N}$ 😒
**Stochastic Telegrapher's eqns. (single line):**

\[
\frac{d}{dz} V(z, s, \beta) = -Z(s, \beta) I(z, s, \beta),
\]
\[
\frac{d}{dz} I(z, s, \beta) = -Y(s, \beta) V(z, s, \beta)
\]

- \(V\) and \(I\): unknown voltage and current along the line
  - function of position, frequency and of stochastic parameter \(\beta\)
- \(s = j\omega\); \(Z = R + sL\) and \(Y = G + sC\) i.e. known p.u.l. TL parameters
- assume – by way of example - that \(\beta\) is a Gaussian random variable:

\[
\beta = \mu_\beta(1 + \sigma_\beta \xi)
\]

normalized Gaussian random variable with zero mean and unit variance
step 1: Hermite “Polynomial Chaos” expansion of Telegrapher’s eqns.:

\[
\frac{d}{dz} V(z, s, \beta) = -Z(s, \beta) I(z, s, \beta)
\]

\[
V(z, s, \beta) = \sum_{k=0}^{K} V_k(z, s) \phi_k(\xi)
\]

\[
Z(s, \beta) = \sum_{k=0}^{K} Z_k(s) \phi_k(\xi)
\]

\[
I(z, s, \beta) = \sum_{k=0}^{K} I_k(z, s) \phi_k(\xi)
\]

inner product:

our Gaussian distribution

\[
< f(\xi), g(\xi) > = \int_{-\infty}^{+\infty} f(\xi) g(\xi) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \xi^2} d\xi
\]

“judiciously” selected inner product such that

\[
< \phi_k(\xi), \phi_m(\xi) > = k! \delta_{km}
\]
- **expanded TL equations**

\[
\frac{d}{dz} \sum_{k=0}^{K} V_k(z, s) \phi_k(\xi) = - \sum_{k=0}^{K} \sum_{l=0}^{K} Z_k(s) I_l(z, s) \phi_k(\xi) \phi_l(\xi)
\]

\[
\frac{d}{dz} \sum_{k=0}^{K} I_k(z, s) \phi_k(\xi) = - \sum_{k=0}^{K} \sum_{l=0}^{K} Y_k(s) V_l(z, s) \phi_k(\xi) \phi_l(\xi)
\]

- **step 2: Galerkin projection on the Hermite polynomials** \( \phi_m(\xi), m = 0, \ldots, K \)

\( \forall m = 0, \ldots, K, \)

\[
\frac{d}{dz} V_m(z, s) = - \sum_{k=0}^{K} \sum_{l=0}^{K} \alpha_{klm} Z_k(s) I_l(z, s)
\]

\[
\frac{d}{dz} I_m(z, s) = - \sum_{k=0}^{K} \sum_{l=0}^{K} \alpha_{klm} Y_k(s) V_l(z, s)
\]

\( \alpha_{klm} = \langle \phi_k(\xi) \phi_l(\xi), \phi_m(\xi) > / m! \)

"augmented" set of deterministic TL eqns.

(\( \beta \) has been eliminated)

+ 😊 deterministic
+ 😊 solution yields complete statistics, i.e. mean, standard dev., skew, ..., PDF
+ 😊 again (coupled) TL- equations
+ 😦 larger set (K times the original)
+ 😊 still much faster than Monte Carlo
- $\beta$: Gaussian RV: $\mu_\beta = 2$ mm and $\sigma_\beta = 10\%$  
  $\zeta$: Gaussian RV: $\mu_\zeta = 3$ mm and $\sigma_\zeta = 8\%$

- transfer function: $H(s) = \frac{V_1(s)}{E(s)}$  
  (ii) forward crosstalk $FX(s) = \frac{V_2(s)}{E(s)}$

- compare with Monte Carlo run (50000 samples)

- efficiency of the Galerkin Polynomial Chaos

<table>
<thead>
<tr>
<th>Technique</th>
<th>CPU time [s]</th>
<th>Speed-up factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>setup</td>
<td>solve</td>
</tr>
<tr>
<td>Novel approach</td>
<td>0.23</td>
<td>8.09</td>
</tr>
<tr>
<td>Monte Carlo</td>
<td>1895.72</td>
<td></td>
</tr>
</tbody>
</table>
Transfer function $H(s, \beta, \zeta)$

Forward crosstalk $FX(s, \beta, \zeta)$

- **full-lines:** mean values $\mu$ using SGM
- **circles:** mean values $\mu$ using MC
- **dashed lines:** $\pm 3\sigma$-variations using SGM
- **squares:** $\pm 3\sigma$-variations using MC

**gray lines:** MC samples

Frequency $f$ [GHz]
- introduction
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  – PART 2: the overall approach
- brief conclusions
- questions & discussion
So far:

- tractable variability analysis of (on-chip) interconnects
  (and passive multiports) outperforming Monte Carlo analysis
- relies on Matlab implementations of the presented techniques
- only relatively small passive circuits with few random variables

Next:

- extension of techniques to include nonlinear and active devices
- extension to many randomly varying parameters
- integration into SPICE-like design environments
  - perform transient analyses
  - simulate complex circuit topologies including connectors, via’s, packages, drivers, receivers
remember – slide 35 - PC projection and testing results in:

“augmented” set of deterministic TL eqns. → can be directly imported in SPICE

random substrate thickness, permittivity and loss tangent

HSPICE Monte Carlo (1000 runs): ... 38 min
HSPICE polynomial chaos: .................. 7 s
Speed-up: ..................................... 310 x
Linear terminations

\[ i(t, \xi) = C \frac{d}{dt} v(t, \xi) \rightarrow \text{random variable } \xi \]

\[ i_0 \varphi_0 + i_1 \varphi_1 + \cdots = C \left( \frac{d}{dt} v_0 \varphi_0 + \frac{d}{dt} v_1 \varphi_1 + \cdots \right) \rightarrow \text{PC expansion} \]

\[ i_m(t) = C \frac{d}{dt} v_m(t) \rightarrow \text{decoupled equations after projection} \]

the deterministic augmented lines share the same termination:
Non-linear terminations

the deterministic line \( m \) now has
the following termination:

\[ j_1 a_{m1} w_1 + j_2 a_{m2} w_2 + \ldots \]

voltage controlled \((v_{L0}, v_{L1}, \ldots)\)
current source

applicable to

- arbitrary device models
- transistor-level descriptions
- behavioral macromodels
- encrypted library models
Non-linear terminations

the deterministic line \( m \) now has
the following termination:

\[
   i_{Lm} = \langle F(v_{L0}\varphi_0 + v_{L1}\varphi_1 + \ldots, \varphi_m), F(v_{Lm}) \rangle
\]

\text{but} \quad i_{Lm} = \int F(v_{L0}\varphi_0(\xi) + v_{L1}\varphi_1(\xi) + \ldots) \varphi_m(\xi) w(\xi) d\xi

\approx \sum_{q=1}^{Q} F(v_{L0}a_{0q} + v_{L1}a_{1q} + \ldots) a_{mq} w_q = j_q

\]

\[ a_{mq} \triangleq \varphi_m(\xi_q) \]

\( \xi_q \): quadrature nodes

\( w_q \): quadrature weights

applicable to

\begin{itemize}
  \item arbitrary device models
  \item transistor-level descriptions
  \item behavioral macromodels
  \item encrypted library models
\end{itemize}

voltage controlled \( (v_{L0}, v_{L1}, \ldots) \) current source
16-bit digital transmission channel

random power rail resistance and package parasitics

Monte Carlo (1k runs) .......................... ~ 2.3 days
Polyn. chaos (Galerkin-based) ................. ~ 9 h
Polyn. chaos (stochastic testing-based) ... ~ 1 h 🎶
Overall speed-up ............................................. 47 x
25 random variables using a point-matching technique:

- parasitic R’s, L’s and C’s of BJT (10%)
- forward current gain (10%)
- ∀ lumped components in LNA schematic (10%)
- widths of 4 transmission lines (5%)

for the same accuracy $10^5$ Monte Carlo single circuit simulations are needed versus only 351 for the new technique speed-up factor: 285
Conclusions

- broad classes of coupled multiconductor transmission lines (MTLs) can be handled;
- efficient and accurate RLGC modelling of MTLs from DC to skin-effect regime is possible thanks to the differential surface current concept;
- MTL variations along the signal propagation direction can be efficiently dealt with thanks to a 2-step perturbation technique;
- all frequency and time-domain statistical signal data can be efficiently collected for many random variations both in MTL characteristics and in linear and non-linear drivers, loads, amplifiers, … thanks to advanced Polynomial Chaos approaches – by far outperforming Monte Carlo methods;
- for very many random variables the curse of dimensionality remains cfr. roughness analysis or scattering problems → ongoing research;
- initial statistics can be very hard to get e.g. a multipins connector → ongoing research.
Thanks to all PhD students and colleagues of the EM group I have been working with on these topics over very many years:

- Niels Faché (now with Keysight Technologies - USA)
- Jan Van Hese (now with Keysight Technologies - Belgium)
- F. Olyslager (full professor at INTEC, UGent – deceased)
- Thomas Demeester (post-doc at INTEC, UGent)
- Luc Knockaert (assistant professor at INTEC, UGent)
- Tom Dhaene (full professor at INTEC, UGent)
- Dries Vande Ginste (full professor at INTEC, UGent)
- Hendrik Rogier (full professor at INTEC, UGent)
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Close collaboration on statistical topics with
Prof. Flavio Canavero (EMC Group, Dipartimento di Elettronica, Politecnico di Torino (POLITO), Italy)
Thank you for your attention!!

- additional reading material: see included list *restricted* to our own work

Questions and Discussion?

- additional questions: right now or at daniel.dezutter@ugent.be
Bibliographic references D. De Zutter et al.

Differential admittance R,L,G,C- modelling


Multiconductor Transmission Line perturbation technique


Variability analysis of interconnects


